WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
QUESTIONS OF REVIEW 2: Functions

Name: Answers.

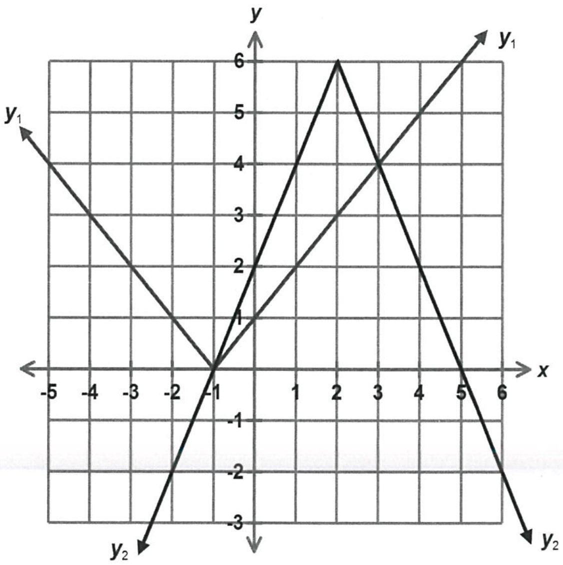
Wednesday 29th March Time: 40 minutes Mark /35

Calculator free.

Average 30.

1. [3 & 3 = 6 marks]

The graphs of y_1 and y_2 are shown on axes to the right.



(a) Use the graph to solve the following equations.

(i) $y_1 = 3$

$x = -4$ or 2

(ii) $y_2 \geq 0$

$-1 \leq x \leq 5$

(iii) $y_2 < y_1$

$x < -1$ or $x > 3$

(b) State the equation for the graph of

(i) y_1 $= |x+1|$

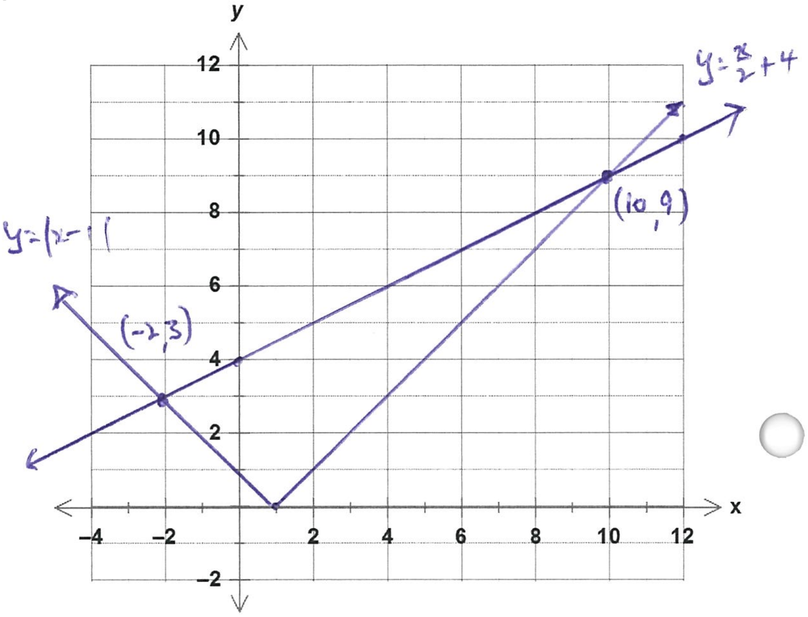
(ii) y_2 $= 6 - 2|x-2|$

$= 6 - |2x-4|$

2. **[5 marks]**

Calculate where $y = |x-1|$ intersects $y = \dfrac{x}{2} + 4$.

Represent your solution on the axes provided.



$x \geq 1$ $\quad x - 1 = \frac{x}{2} + 4$

$\therefore \frac{x}{2} = 5$

$\therefore x = 10$

$x < 1$ $\quad -x + 1 = \frac{x}{2} + 4$

$\therefore \frac{3x}{2} = -3$

$\therefore x = -2$

ie $(-2, 3)$ & $(10, 9)$

3. **[5 marks]**

$f(x) = |x|$ and $g(x) = |x+2|$

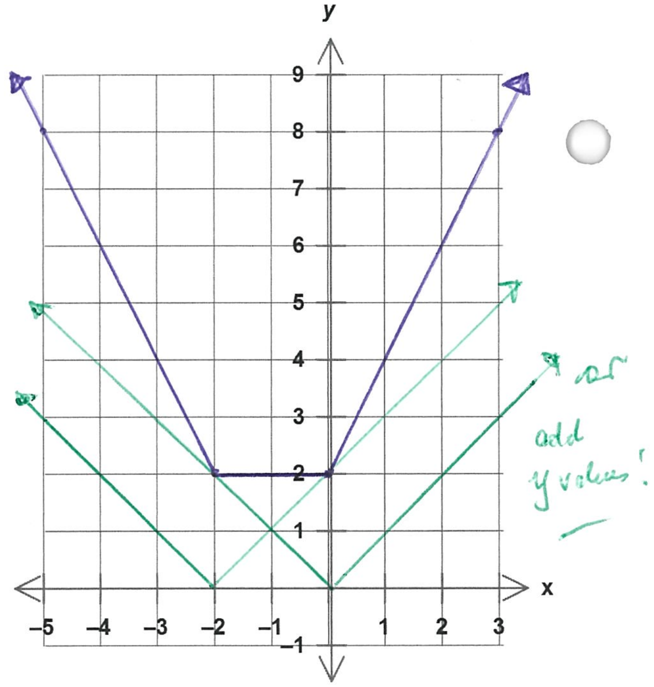
Determine a piecewise defined expression for the sum $f(x) + g(x)$ and sketch $y = f(x) + g(x)$ on these axes.

$x \geq 0 \quad f(x) + g(x) = x + x + 2 = 2x + 2$

$-2 < x < 0 \quad f(x) + g(x) = -x + x + 2 = 2$

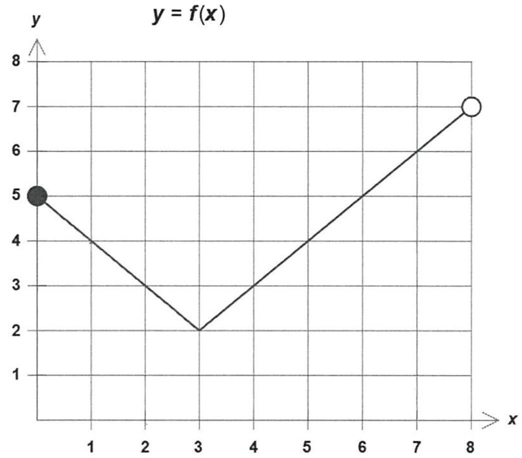
$x \leq -2 \quad f(x) + g(x) = -x - x - 2 = -2x - 2$

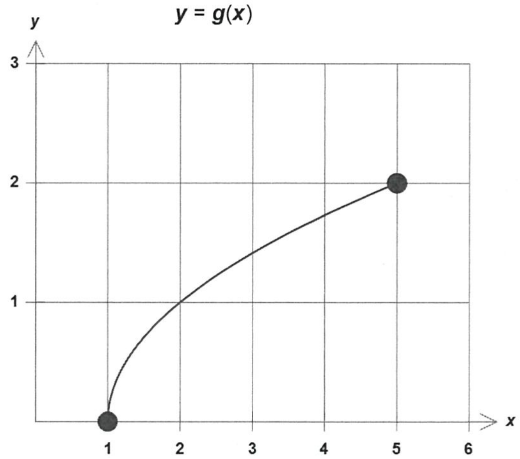
$$\therefore f(x) + g(x) = \begin{cases} 2x + 2 & x \geq 0 \\ 2 & -2 \leq x < 0 \\ -2x - 2 & x \leq -2 \end{cases}$$



4. **[2, 2 & 6 = 10 marks]**

The graphs of $y = f(x)$ and $y = g(x)$ are shown.





(a) Does $f(x)$ possess an inverse function? Explain

No, it is not 1-1

(b) Find

(i) $g \circ f(3)$

$= g(2) = 1$

(ii) $f \circ g(5)$

$= f(2) = 3$

(c) State

(i) the domain of g

$\mathbb{R},\ 1 \le x \le 5$

(ii) the range of f

$\mathbb{R},\ 2 \le y < 7$

(iii) the maximal range of $f \circ g(x)$

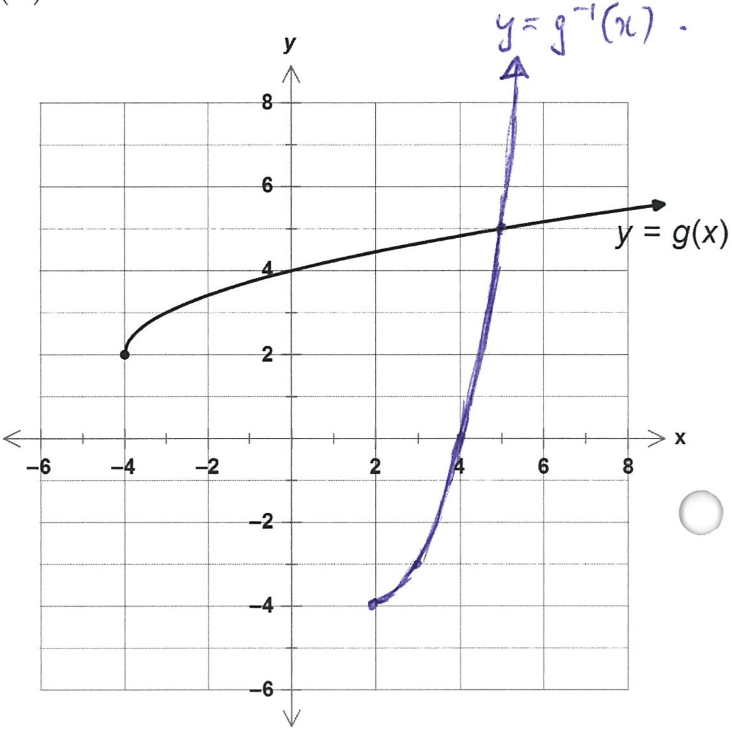
$\mathbb{R},\ 3 \le y \le 5$

(iv) the maximal domain of $g \circ f(x)$

$\mathbb{R},\ 0 \le x \le 6$

5. **[2, 2, 2, 1 & 2 = 9 marks]**

The axes to the right show the graph of $g(x)=\sqrt{x+4}+2$.



(a) Find the value of $(g\circ f)(1)$ if $f(x)=2x-5$.

$= g(-3)$

$= 3$

(b) **(i)** State the range of $g^{-1}(x)$

$-4 \leq y$

or $y \geq -4$

(ii) State the domain of $g^{-1}(x)$

$x \geq 2$

(c) Find the defining rule for $g^{-1}(x)$ in simplest form.

$x = \sqrt{y+4} + 2$ for $x \geq 2$, $y \geq -4$

$\sqrt{y+4} = x - 2$

$y + 4 = (x-2)^2$

$y = (x-2)^2 - 4 = x^2 - 4x$ for $x \geq 2$

(d) Is $g^{-1}(x)$ one-to-one?

Yes, over $x \geq 2$

(e) On the axes above, add a sketch of the graph of $y=g^{-1}(x)$ showing the coordinates of all relevant features clearly.